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Wear Consideration in Gear Design for Space Applications

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WEAR CONSIDERATION IN GEAR DESIGN FOR SPACE APPLICATIONS

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ABSTRACT

A procedure is described that was developed for evaluating the wear in a set of gears in mesh under high load and low rotational speed. The method can be used for any low-speed gear application, with nearly negligible oil film thickness, and is especially useful in space stepping mechanism applications where determination of pointing error due to wear is important, such as in long life sensor antenna drives.

A method is developed for total wear depth at the ends of the line of action using a very simple formula with the slide to roll ratio V_s/V_r. A method is also developed that uses the wear results to calculate the transmission error also known as pointing error of a gear mesh.

INTRODUCTION

Gears used in space applications are usually thought of as "instrument gears" with negligible load that must maintain position accuracy throughout their operational life or as deployment/actuator gears that may have inertial loads but a very low number of duty cycles.

More and more frequently the combined instrument/power drives are emerging on the space mechanisms scheme such as drives for weather satellites where pointing accuracy (as in instruments) as well as substantial inertial load are imposed due to momentum compensated antennas. This type of drive may also have a continuous duty cycle for up to 10 years. Such drives cannot be corrected by use of anti-backlash devices because this contributes to more wear as a result of the additional anti-backlash device loading springs. Also "position accuracy" is not corrected by such devices that are still subjected to severe wear.

A gear arrangement must be devised that will minimize wear effects due to gear ratio and provide addendum modification that will minimize the tooth wear rate. This paper describes a method that has been developed for analyzing the total gear tooth wear and its effect on pointing accuracy. The results can also be used to analyze the resulting effect on gear dynamic loading.

APPROACH TO SOLUTION

Before describing a detailed analytical approach, it is necessary to understand the background assumptions and analysis conditions assumed. First of all there are three lubrications regimes (Peterson, 1980) in geared drives: (1) Boundary (all asperity contact - no film), (2) mixed, and (3) full film (no asperity contact). This paper will deal with wear in the boundary lubrication regime only. Wear in the full film lubrication regime is negligible.

Second it is presumed that people designing gears for space applications will have a gear design computer program from which parameters can be pulled for use in the analysis presented herein. This saves considerable time and allows use of the simplified method presented at the end of this paper.

The conventional approach to wear (Dudley, 1962 and Akin, 1973) is to consider that wear can consist of one or more of the following modes (1) low speed lubricated wear, (2) pitting initiated wear, (3) abrasive wear, and (4) scuffing or scoring that my eventually lead to seizure. Only lubricated low speed adhesive wear is considered in this paper. The sections to follow provide a quantitative analysis of low speed adhesive wear in gears. Although spur gears are used here as the example case, the method presented would be applicable to other types of gears.

DEVELOPMENT OF THE THEORY

The boundary lubrication regime is defined quantitatively as gears operating at a speed and lubricant viscosity such that the lambda ratio (λ) is less than 0.4 ($0 < \lambda \leq 0.4$) where:

$$\lambda = \frac{\text{Lubricant EHD film thickness}}{\text{Composite Asperital RMS height}} \tag{1}$$

This allows the assumption that substantial asperity interaction exists as is the case.

Realizing that the substance of wear in gear action is due to sliding along the line of action, as shown in Fig. 1, the slide-to-roll ratio (also known as specific

sliding) and the compressive stress must be calculated. The necessary formulas may be found in Gay (1970) and other references, to determine the sliding distance per pass at any point on the tooth profile. The Archard wear equation (Rabinowicz, 1965) will be used to calculate depth wear (h) as follows

$$h = \frac{KW_N X}{3pA_a} \quad (2)$$

where

K wear coefficient (dimensionless)

W_N load normal to the tooth (along line of action)

X total sliding distance (for total life)

p flow pressure of substrate (hardness)

A_a apparent area of contact (Hertzian area)

The wear coefficient (K) is determined experimentally but usually falls within the range of 3×10^{-8} to 6.7×10^{-5} for lubricated gears and as high as 1.7×10^{-3} for clean unlubricated surfaces (Peterson, 1980) in an earth environment.

The normal load W_N is usually calculated from the applied torque (T) as follows

$$W_N = \frac{T}{R_b} = \frac{T}{R \cos \phi} \quad (3)$$

where

R_b is the involute base circle radius

R pitch radius

ϕ pressure angle (transverse)

Determination of the total sliding distance (x) past a specific point of contact is the most important calculation to be made. It can be determined as follows

$$x = V_{ss} b C_y \quad (4)$$

where

V_{ss} slide-to-roll ratio = V_s/V_r determined at the end of action where it is a maximum

V_s $V_{r1} - V_{r2}$ = sliding velocity

V_r $\omega\rho$ = rolling velocity

ρ R sin ϕ_i = radius of curvature at point i

ω rotational velocity, rad/sec

ϕ_i pressure angle at instantaneous point of contact i

b width of Hertzian band of contact

C_y total number of contact cycles (life)

The slide-to-roll ratio V_{ss} may be determined from equations in Khiralla (1976) or from a Gear Design Computer program.

The Hertzian band of contact (b) may be determined by the method of Dudley (1954 or 1984):

$$b = \left[\frac{16W_N\rho}{FE'\left(\frac{\pi}{1 - v^2}\right)}\right]^{1/2} \quad (5)$$

where

$\rho = \frac{\rho_p \rho_g}{\rho_p + \rho_g}$ = relative radius of curvature

F = face width gear

$E' = \frac{E_1 E_2}{E_1 + E_2}$ and when $E_1 = E_2$: $E' = E/2$

v = Poisson's ratio

The contact compressive stress can be used to calculate b as follows:

$$b = \frac{3.63\, S_c \rho}{E'} \text{ (for steel gears only)} \quad (6)$$

where

$S_c = \left[\frac{0.35\, W_N E'}{F\rho}\right]^{1/2}$ = compressive stress (AGMA, 1982)

The flow pressure or hardness can be approximated by:

$$p = 1500\ BH (psi)$$

where

BH Brinell hardness number

$$BH = \left(\frac{1585}{122 - R_c}\right)^2$$

Then the apparent area of contact can be calculated from

$$A_a = b\,F = \text{Herzian contact area} \quad (7)$$

where: F = gear face width, in combining Eqs. (2), (4), and (7), the wear formula can be presented as:

$h = \frac{KW_N(V_{ss}bC_y)}{3PbF}$ so that the b's can be cancelled

resulting in the formula:

$$h = \frac{KW_N V_{ss} C_y}{3PF} \quad (8)$$

providing a very simple formula with only on variable (V_{ss}) as a function of position on the line of action.

In practice a gear computer program may be used to calculate the V_{ss} values at the extreme positions on the line of action as shown in Table I. The normal

wear on the pinion must be added to the wear on the gear to obtain the total wear at a specific point on the line of action.

Thus, if we observe in Eq. (8) that the grouped parameters

$\frac{KW_N}{3PF}$ form a constant for a given gear set and that the total wear can be formulated as:

$$\Sigma h_1 = h_{p1} + h_{g1} = \frac{KW_N}{3PF}\left(V_{ss1}C_{y1} + V_{ss2}C_{y2}\right) \qquad (9)$$

Further, the contact cycles for the pinion can be calculated from:

$$C_{y1} = m_g\, C_{y2}$$

where

m_g gear ratio. Thus:

$$\Sigma h_1 = \frac{KW_N C_{y2}}{3PF}\left(m_g V_{ssp1} + V_{ssg1}\right)_1 \text{ at OD of gear, and}$$

$$\Sigma h_5 = \frac{KW_N C_{y2}}{3PF}\left(m_g V_{ssp2} + V_{ssg2}\right)_5 \text{ at OD of pinion.}$$

The task now is to balance the wear at the extreme ends of the line of action (see Fig. 2) so that $\Sigma h_1 = \Sigma h_5$. This can be accomplished by adjusting the addendum of the pinion and gear on standard centers. Thus $\Delta a_p = -\Delta a_g$ is adjusted until:

$$\Sigma V_1 = (m_g V_{ssp1} + V_{ssg1})_1 = (m_g V_{ssp2} + V_{ssg2})_5 = \Sigma V_5 \qquad (10)$$

for the opposite ends of the line of action where

Δa_p addendum modification on pinion = $\Delta N_p/2P_d$

Δa_g addendum modification on gear = $\Delta N_g/2P_d$

$\Delta N_{p,g}$ virtual change in number of teeth

P_d diametral pitch

Therefore, the only special data needed for inputs would be ΔN_p and ΔN_g and the outside diameters d_O and D_O calculated from

$$d_o = \frac{N_p + 2 + \Delta N_p}{P_d}$$

and

$$D_o = \frac{N_g + 2 - \Delta N_g}{P_d} \quad \text{both on standard centers.}$$

Finally, as an end result the designer would want to know the change in pointing accuracy as a result of wear. This can be calculated from:

$$e_1 = \frac{\Sigma h_1}{R_b} \text{ rad} \qquad (11)$$

or

$$e_1 = \frac{180\ \Sigma h_1}{\pi R_b} \text{ degrees, and} \qquad (12)$$

$$e_5 = \frac{\Sigma h_5}{R_b} \text{ rad}$$

or

$$e_5 = \frac{180\ \Sigma h_5}{\pi R_b} \text{ degrees, so that by design} \qquad (13)$$

$$e_1 \doteq e_5$$

If Eq. (10) is not true, the addendum modifications $\Delta a_p = -\Delta a_g$ are iterated until Eq. (10) is true within an acceptable tolerance (say 1 percent).

A SPECIFIC EXAMPLE

Let

Diametral Pitch $P_d = 48$

Number of teeth in pinion $N_p = 24$

Pressure angle $\phi = 20°$

Number of teeth in gear $N_g = 120$

Gear base radius $R_b = 1.1746$

Gear ratio = 5

Addendum modification $\Delta N_p = -\Delta N_g = 0$

Outside diameter of pinion $d_O = 0.54166$

Outside diameter of gear $D_O = 2.54166$

The results of these inputs to a gear computer program provided the output shown in Table 1. The underlined output values V_{ss1} and V_{ss2} are substituted into Eq. (10) as follows

$$\Sigma V_1 = (5|-2.4701| + 0.7118)_1 \neq (5\times.4386 + |-0.7814|)_5 = \Sigma V_5 \quad 13.0623 \neq 2.9744$$

a considerable mismatch results here. The pointing errors due to wear that result here would be (from Eqs. (12) and (13)):

$$e_1 = \Sigma h_1/R_b \quad \text{where} \quad \Sigma h_1 = \frac{KW_N C_{y2}}{3PF}\ \Sigma V_1$$

$$\Sigma h_1 = \frac{5\times10^{-6}\ (11.125)\ 12.6\times10^{-6}}{3 \times 980\ 250 \times 0.125} \times 13.0623 = 0.02491$$

$$e_1 = 0.02491/1.1746 = 0.0212 \text{ rad, or } 1.2°$$

$$e_5 = \Sigma h_5/R_b \quad \text{where} \quad h_5 = \frac{KW_N C_{y2}}{3PF}\ \Sigma V_5 = 0.005671$$

$$e_5 = 0.005671/1.1746 = 0.0048 \text{ rad, or } 0.28°$$

also the top land of the pinion would have been reduced from 0.0144 to 0.0109 and worse the pinion root (LPC) would have been reduced by wear 0.0236 in. (66 percent) of the tooth thickness. After several iterations the final design resulted. Let:

$N_p = 24$, $N_g = 120$, $\Delta N_p = 0.96$, $\Delta N_g = -0.96$

Outside diameter of pinion $d_0 = 0.561666$

Outside diameter of gear $D_0 = 2.521666$

The computer output is shown in Table II. The underlined output values are substituted into Eq. (10) as follows

$$\Sigma V_1 = (5|-0.6725| + 0.4021) \cong (5 \times .5332$$

$$+ |-1.1422|)2 = \Sigma V_5 \quad 3.7646 \cong 3.8082 \text{ within 1 percent.}$$

A good match is provided here. The pointing errors that would result are:

$$e_1 = 0.00718/1.1746 = 0.00611 \text{ rad, or } 0.350°$$

$$e_5 = 0.00726/1.1746 = 0.0618 \text{ rad, or } 0.354°$$

This is a substantial reduction in pointint error e_1 caused by wear from the original standard gear design.

It should be noted here that the above errors e_1 and e_5 are due to wear only and must be added to the original basic error due to backlash and tooth-to-tooth composite errors to get a total pointing error at tne end of life after C_{y1} and C_{y2} cycles of the pinion and gear respectively.

The gear wear life can be the critical failure criteria. As an example, consider an instrument that is considered inoperable if its pointing error exceeds 0.2°. We would calculate its life as follows. Let's say the desired life is 15 000 000 cycles (C_y) over three years and we determine that allowable backlash is

$$e_a = 0.2° \times \pi/180 \times R_b$$

$$e_a = 0.00349 \times 1.1746 = 0.0041 \text{ in. at wear out.}$$

Subtract 0.0018 in. for initial backlash plus tooth-to-tooth composite error (when new) to get 0.0023 in. allowable for wear. Using formula (9) we determine that calculated wear depth h over 3 years is 0.049 in. Thus the predicted wear life will be $0.0023/0.0049 \times 3$ years = 1.4 years or 16.9 months. Since this is less than is desired, a design modification would be necessary.

It should also be noted that the choice of gear arrangement can also be very important in minimizing gear drive pointing error, but this is beyond the scope of this paper.

CLOSURE

The basic tools have been provided in this paper for the minimization of pointing error due to wear through modification of standard gear tooth geometry, also an example has been provided to clearly illustrate the use of the method.

This method is easily adapted to other types of gears using the same principles. This is a first attempt at quantization of gear wear and especially its effect on gear life in terms of an allowable pointing inaccuracy.

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TABLE I. - COMFUTER PRINTOUT FOR SLIDING AND HERTZ STRESS AT 5 POINTS ON THE TOOTH PROFILE

Location on tooth profile (a)	Roll angle, ε, deg	Rolling velocity, in./s		Sliding velocity, in./s		Hertzian stress, q, lb/in.^{2}	Specific sliding ratio	
		V_{r1}	V_{r2}	V_{s1}	V_{s2}		V_{ss1}	V_{ss2}
1	6.82	1.22	4.23	-3.01	3.01	50 808	-2.4701	0.7118
2	17.87	3.20	3.84	-0.64	0.64	57 094	-0.2005	0.1670
3	20.85	3.73	3.73	-0.00	0.00	53 600	0.0000	0.0000
4	21.82	3.90	3.70	0.21	-0.21	52 646	0.0531	-0.0560
5	32.87	5.88	3.30	2.58	-2.58	26 206	0.4386	-0.7814

[a]Code for locations on tooth profile:
1 = Start of Active Profile (SAP)
2 = Lowest Point of Single Tooth Contact (LPSTC)
3 = Pitch point (P)
4 = Highest Point of Single Tooth Contact (HPSTC)
5 = End of Active Profile (EAP)

TABLE II. - COMPUTER PRINTOUT FOR SLIDING AND HERTZ STRESS AT 5 POINTS ON THE TOOTH CONTACT

Location on tooth profile (a)	Roll angle, ε, deg	Rolling velocity, in./s		Sliding velocity, in./s		Hertzian stress, q, lb/in.^{2}	Specific sliding ratio	
		V_{r1}	V_{r2}	V_{s1}	V_{s2}		V_{ss1}	V_{ss2}
1	13.36	2.39	4.00	-1.61	1.61	0	-0.6725	0.4021
2	22.53	4.03	3.67	0.36	-0.36	51 988	0.0892	-0.0980
3	20.85	3.73	3.73	-0.00	0.00	44 858	-0.0000	0.0000
4	28.36	5.07	3.46	1.61	-1.61	47 710	0.3177	-0.4657
5	37.53	6.71	3.13	3.58	-3.58	16 479	0.5332	-1.1422

[a]Code for locations on tooth profile:
1 = Start of Active Profile (SAP)
2 = Lowest Point of Single Tooth Contact (LPSTC)
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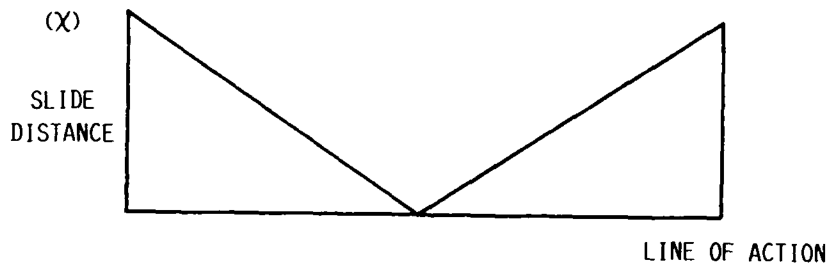

FIGURE 1. - SLIDE DISTANCE ON LINE OF ACTION.

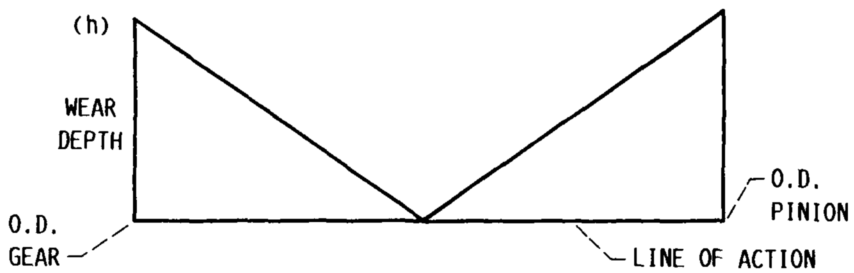

FIGURE 2. - WEAR DEPTH ON LINE OF ACTION.

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16. Abstract

A procedure is described that was developed for evaluating the wear in a set of gears in mesh under high load and low rotational speed. The method can be used for any low-speed gear application, with nearly negligible oil film thickness, and is especially useful in space stepping mechanism applications where determination of pointing error due to wear is important, such as in long life sensor antenna drives. A method is developed for total wear depth at the ends of the line of action using a very simple formula with the slide to roll ratio V_s/V_r. A method is also developed that uses the wear results to calculate the transmission error also known as pointing error of a gear mesh.

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